Year 10 completing the square worksheet

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Name: _____ Date: _____

Question 1

Write $x^2 + 8x + 3$ in the form $(x + h)^2 + k$ by completing the square.

Answer: _____

Question 2

Solve $x^2 - 6x - 3 = 0$ by completing the square. Give exact answers in simplest surd form.

Answer: _____

Question 3

Factorise $x^2 + 6x + 4$ by completing the square.

Answer: _____

Question 4

A rectangle has a length that is 4 cm more than its width. Its area is 30 cm^2. Let the width be w cm.

(a) Show that $w^2 + 4w - 30 = 0$.

(b) Solve the equation by completing the square, giving exact answers.

(c) Use a calculator to find the width and length of the rectangle, correct to two decimal places.

Answer: _____

Question 5

Factorise $x^2 - 10x + 18$ by completing the square.

Answer: _____

Question 6

Solve $x^2 + 4x - 21 = 0$ by completing the square.

Answer: _____

Question 7

Factorise $x^2 + 5x + 2$ by completing the square.

Answer: _____

Question 8

Solve $x^2 - 8x + 7 = 0$ by completing the square.

Answer: _____

Question 9

(a) By completing the square, show that $x^2 + 4x + 9$ cannot be factorised using real numbers.

(b) Factorise $x^2 - 2x - 11$ by completing the square, giving surds in simplest form.

(c) Find all values of k for which $x^2 + 6x + k$ can be factorised using real numbers.

Answer: _____

Question 10

Solve $x^2 + 2x - 5 = 0$ by completing the square. Give your answers correct to 2 decimal places.

Answer: _____

Year 10 completing the square: answers and worked solutions

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Question 1

Key idea: Add and subtract $(b/2)^2$, where b is the coefficient of x.

1. $b/2 = 8/2 = 4$ and $4^2 = 16$. 2. $x^2 + 8x + 3 = x^2 + 8x + 16 - 16 + 3$ 3. $= (x + 4)^2 - 13$ 4. Check: $(x + 4)^2 - 13 = x^2 + 8x + 16 - 13 = x^2 + 8x + 3$ ✓

Answer: $(x + 4)^2 - 13$

Question 2

Key idea: Complete the square on the left side, move the constant across, then take the square root of both sides (remembering ±).

1. $x^2 - 6x + 9 - 9 - 3 = 0$, so $(x - 3)^2 - 12 = 0$. 2. $(x - 3)^2 = 12$ 3. $x - 3 = \pm\sqrt{12} = \pm 2\sqrt{3}$ 4. $x = 3 \pm 2\sqrt{3}$

Answer: $x = 3 + 2\sqrt{3}$ or $x = 3 - 2\sqrt{3}$

Question 3

Key idea: Complete the square to get a difference of two squares, then use $a^2 - b^2 = (a - b)(a + b)$.

1. $(6/2)^2 = 9$, so $x^2 + 6x + 4 = x^2 + 6x + 9 - 9 + 4 = (x + 3)^2 - 5$. 2. Write $5 = (\sqrt{5})^2$: $(x + 3)^2 - (\sqrt{5})^2$. 3. Difference of two squares: $(x + 3 - \sqrt{5})(x + 3 + \sqrt{5})$.

Answer: $(x + 3 - \sqrt{5})(x + 3 + \sqrt{5})$

Question 4

Key idea: Write the area as length × width, rearrange to equal zero, then complete the square. Reject any negative length.

(a) 1. Length = w + 4, so area = $w(w + 4) = 30$. 2. $w^2 + 4w = 30$, so $w^2 + 4w - 30 = 0$.

(b) 1. $w^2 + 4w + 4 - 4 - 30 = 0$, so $(w + 2)^2 = 34$. 2. $w + 2 = \pm\sqrt{34}$ 3. $w = -2 + \sqrt{34}$ or $w = -2 - \sqrt{34}$

(c) 1. $-2 - \sqrt{34}$ is negative, so it is rejected because a width must be positive. 2. $w = -2 + \sqrt{34} = 3.830\ldots \approx 3.83$ cm 3. Length $= w + 4 = 7.830\ldots \approx 7.83$ cm 4. Check: $3.830\ldots \times 7.830\ldots = 30.0$ ✓

Answer: (a) $w(w + 4) = 30$ rearranges to $w^2 + 4w - 30 = 0$ (b) $w = -2 \pm \sqrt{34}$ (c) Width 3.83 cm, length 7.83 cm

Question 5

Key idea: Complete the square, then factorise as a difference of two squares.

1. $(-10/2)^2 = 25$, so $x^2 - 10x + 18 = x^2 - 10x + 25 - 25 + 18 = (x - 5)^2 - 7$. 2. $(x - 5)^2 - (\sqrt{7})^2 = (x - 5 - \sqrt{7})(x - 5 + \sqrt{7})$

Answer: $(x - 5 - \sqrt{7})(x - 5 + \sqrt{7})$

Question 6

Key idea: Complete the square, then take the square root of both sides.

1. $x^2 + 4x = 21$, so $(x + 2)^2 = 21 + 4 = 25$. 2. $x + 2 = \pm 5$. 3. $x = 3$ or $x = -7$.

Answer: $x = 3$ or $x = -7$

Question 7

Key idea: When the coefficient of x is odd, $(b/2)^2$ is a fraction. Work carefully with the fractions.

1. $(5/2)^2 = 25/4$, so $x^2 + 5x + 2 = (x + 5/2)^2 - 25/4 + 8/4 = (x + 5/2)^2 - 17/4$ 2. $17/4 = ((\sqrt{(17)})/(2))^2$ 3. Difference of two squares: $(x + 5/2 - (\sqrt{(17)})/(2))(x + 5/2 + (\sqrt{(17)})/(2))$

Answer: $(x + 5/2 - (\sqrt{(17)})/(2))(x + 5/2 + (\sqrt{(17)})/(2))$

Question 8

Key idea: Complete the square, then take the square root of both sides.

1. $x^2 - 8x = -7$, so $(x - 4)^2 = -7 + 16 = 9$. 2. $x - 4 = \pm 3$. 3. $x = 7$ or $x = 1$.

Answer: $x = 7$ or $x = 1$

Question 9

Key idea: After completing the square, $(x + h)^2 - m$ factorises over the real numbers only when $m \geq 0$, because then m is a perfect square of a real number.

(a) 1. $x^2 + 4x + 9 = x^2 + 4x + 4 - 4 + 9 = (x + 2)^2 + 5$ 2. This is a sum of two squares, not a difference, and $(x + 2)^2 + 5 \geq 5 > 0$ for all x. 3. So it cannot be written as a product of real linear factors.

(b) 1. $x^2 - 2x - 11 = x^2 - 2x + 1 - 1 - 11 = (x - 1)^2 - 12$ 2. $\sqrt{(12)} = 2\sqrt{3}$, so $(x - 1)^2 - (2\sqrt{3})^2$. 3. $= (x - 1 - 2\sqrt{3})(x - 1 + 2\sqrt{3})$

(c) 1. $x^2 + 6x + k = (x + 3)^2 - 9 + k = (x + 3)^2 - (9 - k)$ 2. This is a difference of squares when $9 - k \geq 0$, that is $k \leq 9$.

Answer: (a) $x^2 + 4x + 9 = (x + 2)^2 + 5$, a sum of squares that is always positive, so it has no real factors (b) $(x - 1 - 2\sqrt{3})(x - 1 + 2\sqrt{3})$ (c) $k \leq 9$

Question 10

Key idea: Complete the square, take square roots, then round at the end.

1. $x^2 + 2x = 5$, so $(x + 1)^2 = 6$. 2. $x + 1 = \pm\sqrt{6}$, so $x = -1 \pm \sqrt{6}$. 3. $x \approx 1.45$ or $x \approx -3.45$.

Answer: $x \approx 1.45$ or $x \approx -3.45$