Year 10A factorising worksheet

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Name: _____ Date: _____

Question 1

Factorise $6x^3 + 9x^2 - 3x$.

Answer: _____

Question 2

Factorise $x^2 + 7x + 12$.

Answer: _____

Question 3

Factorise $6x^2 + 11x + 3$.

Answer: _____

Question 4

Factorise $-12a^3b^2 + 8a^2b^3 - 4a^2b$.

Answer: _____

Question 5

Factorise each expression.

(a) $x^2 - 49$

(b) $x^2 - 10x + 25$

Answer: _____

Question 6

Solve $4x^2 - 4x - 3 = 0$.

Answer: _____

Question 7

A rectangular garden bed has an area that can be expressed as $5x(x+3) - 2(x+3)$ square metres, where x is a positive number. Write the area as a product of two factors, and find the area when $x = 4$.

Answer: _____

Question 8

A rectangular garden has an area of $x^2 + 11x + 28$ square metres, where x is a positive number. The length of the garden is $(x + 7)$ metres. Write an expression for the width of the garden by factorising the area expression, then find the width when $x = 3$.

Answer: _____

Question 9

A rectangular garden has a length that is 3 metres more than twice its width. The area of the garden is 27 square metres. Find the width of the garden.

Answer: _____

Question 10

Factorise $12a^3 - 8a^2$ by taking out the highest common factor.

Answer: _____

Year 10A factorising: answers and worked solutions

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Question 1

Key idea: Find the HCF of all terms, write it outside the brackets, then divide each term by the HCF to find what goes inside.

1. Find the HCF of the coefficients 6, 9 and 3: the HCF is 3. 2. Find the HCF of x^3, x^2 and x: the lowest power is $x^1 = x$. 3. So the overall HCF is 3x. 4. Divide each term by 3x: ($6x^3$)/(3x) = $2x^2$, ($9x^2$)/(3x) = 3x, -3x/3x = -1 5. Write the factorised form: $6x^3 + 9x^2 - 3x$ = $3x(2x^2 + 3x - 1)$

Check: $3x(2x^2 + 3x - 1) = 6x^3 + 9x^2 - 3x$ ✓

Answer: $3x(2x^2 + 3x - 1)$

Question 2

Key idea: Find two integers that multiply to 12 and add to 7.

1. List factor pairs of 12: (1, 12), (2, 6), (3, 4). 2. Check which pair adds to 7: 3 + 4 = 7. ✓ 3. Write the factorised form: (x + 3)(x + 4). 4. **Verify by expanding:** (x + 3)(x + 4) = $x^2 + 4x + 3x + 12 = x^2 + 7x + 12$. ✓

Answer: (x + 3)(x + 4)

Question 3

Key idea: For a non-monic quadratic $ax^2 + bx + c$, find two numbers that multiply to ac and add to b, then split the middle term and factorise by grouping.

1. Identify a = 6, b = 11, c = 3, so ac = 6 × 3 = 18. 2. Find two numbers that multiply to 18 and add to 11: these are 2 and 9. 3. Split the middle term: $6x^2 + 2x + 9x + 3$. 4. Factorise by grouping in pairs: 2x(3x + 1) + 3(3x + 1). 5. Factor out the common bracket: (3x + 1)(2x + 3).

Answer: (3x + 1)(2x + 3)

Question 4

Key idea: When the leading term is negative, take out a negative common factor so the first term inside the brackets is positive. Find the full HCF including all variables.

1. Find the HCF of the coefficients 12, 8 and 4: the HCF is 4. 2. Include the negative sign: take out -4. 3. Find the HCF of the variable parts: lowest power of a is a^2; lowest power of b is $b^1 = b$. So the variable HCF is a^2b. 4. Overall factor to take out: $-4a^2b$. 5. Divide e

ach term by $-4a^2b$: $(-12a^3b^2)/(-4a^2b) = 3ab$, $(8a^2b^3)/(-4a^2b) = -2b^2$, $(-4a^2b)/(-4a^2b) = 1$ 6. W rite the factorised form: $-12a^3b^2 + 8a^2b^3 - 4a^2b = -4a^2b(3ab - 2b^2 + 1)$

Check: $-4a^2b(3ab - 2b^2 + 1) = -12a^3b^2 + 8a^2b^3 - 4a^2b$ ✓

Answer: $-4a^2b(3ab - 2b^2 + 1)$

Question 5

Key idea: Recognise special quadratic forms: $a^2 - b^2 = (a-b)(a+b)$ and $a^2 - 2ab + b^2 = (a - b)^2$.

Part (a) 1. Write $x^2 - 49$ as $x^2 - 7^2$. 2. Apply the difference of two squares rule: $(x - 7)(x + 7)$.

Part (b) 1. Check: $\sqrt{(25)} = 5$ and the middle term $-10x = -2 \times x \times 5$. ✓ 2. This is a perfect-square trinomial: $(x - 5)^2$. 3. **Verify:** $(x-5)^2 = x^2 - 10x + 25$. ✓

Answer: (a) $(x - 7)(x + 7)$ (b) $(x - 5)^2$

Question 6

Key idea: Factorise the left-hand side using the ac method, then apply the null factor law.

1. Identify $a = 4$, $b = -4$, $c = -3$, so $ac = 4 \times (-3) = -12$. 2. Find two numbers that multiply to -12 and add to -4: these are -6 and 2. 3. Split and group: $4x^2 - 6x + 2x - 3 = 2x(2x - 3) + 1(2x - 3) = (2x - 3)(2x + 1)$. 4. Apply the null factor law: $2x - 3 = 0$ or $2x + 1 = 0$. 5. Solve each: $x = 3/2$ or $x = -½$

Answer: $x = 3/2$ or $x = -½$

Question 7

Key idea: When the same bracket appears in every term, the whole bracket is a common factor and can be taken out just like a single term.

1. Identify the common binomial factor: $(x + 3)$ appears in both terms. 2. Take out $(x + 3)$: $5x(x+3) - 2(x+3) = (x+3)(5x-2)$ 3. Check by expanding: $(x+3)(5x-2) = 5x^2 - 2x + 15x - 6 = 5x^2 + 13x - 6$, and $5x(x+3) - 2(x+3) = 5x^2 + 15x - 2x - 6 = 5x^2 + 13x - 6$ ✓ 4. Substitute $x = 4$: Area $= (4+3)(5 \times 4 - 2) = 7 \times 18 = 126$ square metres

Answer: $(x+3)(5x-2)$; the area when $x = 4$ is 126 square metres.

Question 8

Key idea: Area of a rectangle equals length multiplied by width, so width equals area divided by length. Factorise the area expression first.

1. Find two integers that multiply to 28 and add to 11: $4 + 7 = 11$ and $4 \times 7 = 28$. ✓ 2. Factorise: $x^2 + 11x + 28 = (x + 4)(x + 7)$. 3. Width $= ((x + 4)(x + 7))/((x + 7)) = x + 4$ metres. 4. Substitute $x = 3$: width $= 3 + 4 = 7$ metres.

Answer: Width = (x + 4) metres; when x = 3, the width is 7 metres.

Question 9

Key idea: Write an equation for the area using the given information, rearrange to standard form, then solve by factorising.

1. Let the width be w metres. Then the length is (2w + 3) metres. 2. Write the area equation: w(2w + 3) = 27, which expands to $2w^2 + 3w = 27$. 3. Rearrange to standard form: $2w^2 + 3w - 27 = 0$. 4. Use the ac method: ac = 2 × (-27) = -54. Two numbers that multiply to -54 and add to 3: these are 9 and -6. Split and group: $2w^2 + 9w - 6w - 27 = w(2w + 9) - 3(2w + 9) = (2w + 9)(w - 3)$. 5. Apply the null factor law: 2w + 9 = 0 gives w = -9/2 (reject, as width must be positive), or w - 3 = 0 gives w = 3.

Answer: The width of the garden is 3 metres.

Question 10

Key idea: The HCF of all terms is taken outside the brackets, and each term is divided by it.

1. Find the HCF of the coefficients: HCF(12, 8) = 4. 2. Find the HCF of the variable parts: the lowest power of a in both terms is a^2. 3. So the overall HCF is $4a^2$. 4. Divide each term by $4a^2$: $12a^3 \div 4a^2 = 3a$ and $8a^2 \div 4a^2 = 2$. 5. Write in factorised form: $4a^2(3a - 2)$. 6. Check by expanding: $4a^2 \times 3a - 4a^2 \times 2 = 12a^3 - 8a^2$ ✓

Answer: $4a^2(3a - 2)$