Networks: 10 practice questions

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Name: _____ Date: _____

Attempt all questions and show your working. Suggested time: 15 to 20 minutes.

Section A: Consolidating (Questions 1-4)

Question 1

A network has vertices A, B, C and D, and edges AB, AC, BC and CD.

(a) How many vertices and how many edges does it have?

(b) Write down the degree of each vertex.

(c) Find the sum of the degrees. How does it compare with the number of edges?

Question 2

Use the network from Question 1.

(a) Is the network connected? Explain.

(b) Which single edge, if removed, would split the network into two separate parts?

Question 3

A network has 5 vertices with degrees 1, 2, 2, 3 and 2. How many edges does it have?

Question 4

Jade says she has drawn a network with five vertices of degrees 3, 3, 3, 2 and 2. Explain why this is impossible.

Section B: Standard (Questions 5-10)

Question 5

Five towns P, Q, R, S and T are joined by roads PQ, PR, QR, QS, RS, ST and RT.

(a) Draw a table showing the degree of each town.

(b) Explain why it is possible to drive along every road exactly once, but not to finish where you started.

(c) Where must such a drive start and finish?

(d) Write down one such route.

Question 6

The table shows the lengths of roads (in km) between six locations. Roads not listed do not exist, and every road can be travelled in both directions.

Road	AB	AC	BC	BD	CD	CE	DE	DF	EF
Length (km)	4	2	1	5	8	10	2	6	3

Find the shortest route from A to F and its length.

Question 7

A network has vertices A, B, C, D, E and F, and edges AB, BC, CA, DE and EF.

(a) Is the network connected? Explain.

(b) What is the smallest number of edges that must be added to make it connected? Give an example.

Question 8

Euler's formula for a connected planar network is v - e + f = 2, where v is the number of vertices, e the number of edges and f the number of faces (including the outside region).

(a) A connected planar network has 6 vertices and 9 edges. How many faces does it have?

(b) A connected planar network has 8 edges and 5 faces. How many vertices does it have?

Question 9

In a group of six friends, the friendships are: Ava and Ben, Ava and Chloe, Ben and Dev, Chloe and Dev, Dev and Ella, Ella and Finn. This is represented as a network with each person as a vertex.

(a) Who has the most friends in the group?

(b) Find the shortest chain of friendships from Ava to Finn. How many edges does it use?

(c) Name a friendship that, if it ended, would split the group into two parts that are not connected.

Question 10

A school wants to connect five buildings, P, Q, R, S and T, with network cables. The table shows the cost (in thousands of dollars) of each possible cable.

Cable	PQ	PR	QR	QS	RS	RT	ST

Cost ($1000s)	5	7	4	6	3	8	5

(a) Find the cheapest set of cables that connects all five buildings (a minimum spanning tree).

(b) What is the total cost?

(c) How many cables are in your answer? Explain why any spanning tree for 5 buildings has this many edges.

Networks: answers

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Question 1

Answer: (a) 4 vertices, 4 edges (b) A: 2, B: 2, C: 3, D: 1 (c) 8, which is twice the number of edges

Question 2

Answer: (a) Yes: there is a path between every pair of vertices. (b) CD

Question 3

Answer: 5 (sum of degrees 10 ÷ 2)

Question 4

Answer: The sum of the degrees is 13, which is odd, but every edge adds 2 to the sum of degrees, so the sum must be even.

Question 5

Answer: (a) P: 2, Q: 3, R: 4, S: 3, T: 2 (b) Exactly two vertices (Q and S) have odd degree, so there is an Euler trail but no Euler circuit. (c) Start at Q and finish at S, or start at S and finish at Q (d) For example Q, P, R, Q, S, R, T, S (any route using each of the 7 roads once from Q to S or from S to Q)

Question 6

Answer: A, C, B, D, E, F, length 13 km (2 + 1 + 5 + 2 + 3)

Question 7

Answer: (a) No: it has two separate parts (A, B, C and D, E, F), with no path from, for example, A to D. (b) 1, for example CD (any edge joining one of A, B, C to one of D, E, F)

Question 8

Answer: (a) 5 (b) 5

Question 9

Answer: (a) Dev (3 friends) (b) Ava, Ben, Dev, Ella, Finn (or Ava, Chloe, Dev, Ella, Finn): 4 edges (c) Dev and Ella, or Ella and Finn (either is correct)

Question 10

Answer: (a) RS, QR, PQ, ST (b) $17 000 (3 + 4 + 5 + 5 thousand) (c) 4: a tree joining n vertices always has n - 1 edges (each new edge connects one more building, with no cycles).