Counting principles and arrangements: 10 practice questions

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Name: _____ Date: _____

Attempt all questions and show your working. Suggested time: 15 to 20 minutes.

Section A: Consolidating (Questions 1-4)

Question 1

Evaluate:

(a) 5!

(b) (7!)/(5!)

(c) 0!

Question 2

Zoe has 4 shirts, 3 pairs of pants and 2 pairs of shoes. How many different outfits of one shirt, one pair of pants and one pair of shoes can she make?

Question 3

How many different arrangements are there of all the letters of the word MATHS?

Question 4

(a) Write $6 \times 5 \times 4 \times 3 \times 2 \times 1$ using factorial notation.

(b) Write $10 \times 9 \times 8$ as a quotient of two factorials.

Section B: Standard (Questions 5-10)

Question 5

(a) Simplify $(n!)/((n - 1)!)$.

(b) Simplify $((n + 1)!)/((n - 1)!)$.

(c) Solve $(n!)/((n - 2)!) = 30$, where n is a positive whole number.

Question 6

A phone has a 4-digit PIN, using the digits 0 to 9.

(a) How many PINs are possible if digits can be repeated?

(b) How many PINs are possible if no digit can be repeated?

(c) How many PINs have no repeated digit and do not start with 0?

Question 7

Six different books are placed in a row on a shelf. One of them is an atlas and another is a dictionary.

(a) How many different arrangements are there?

(b) How many arrangements have the atlas at the left end?

(c) How many arrangements have the atlas and the dictionary next to each other?

Question 8

Ten runners are in a race. In how many different ways can the gold, silver and bronze medals be awarded? Write your answer as a quotient of factorials and evaluate it.

Question 9

A committee of 3 people is to be chosen from a group of 7 people.

(a) In how many ways can a president, secretary and treasurer be chosen from the 7 people?

(b) Explain why dividing your answer to (a) by 3! gives the number of different committees of 3 people (with no roles).

(c) How many different committees of 3 people are possible?

Question 10

(a) How many arrangements are there of the letters A, B, C, D, E, F if all the letters are different?

(b) Explain why the number of different arrangements of the letters of BANANA is (6!)/(3! × 2!).

(c) Evaluate this number.

Counting principles and arrangements: answers

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Question 1

Answer: (a) 120 (b) 42 (7 × 6) (c) 1

Question 2

Answer: 24 (4 × 3 × 2)

Question 3

Answer: 120 (5!)

Question 4

Answer: (a) 6! (b) (10!)/(7!)

Question 5

Answer: (a) n (b) n(n + 1) (also accept $n^2 + n$) (c) n = 6 (n(n - 1) = 30)

Question 6

Answer: (a) 10,000 (b) 5040 (10 × 9 × 8 × 7) (c) 4536 (9 × 9 × 8 × 7)

Question 7

Answer: (a) 720 (6!) (b) 120 (5!) (c) 240 (2 × 5!, treating the pair as one item that can be arranged 2 ways)

Question 8

Answer: (10!)/(7!) = 720 (10 × 9 × 8)

Question 9

Answer: (a) 210 (7 × 6 × 5) (b) Each committee of 3 people can be given the three roles in 3! = 6 ways, so (a) counts each committee 6 times. (c) 35

Question 10

Answer: (a) 720 (b) If the letters were all different there would be 6! arrangements. Swapping the three A's (3! ways) or the two N's (2! ways) gives the same word, so each arrangement is counted 3! × 2! times. (c) 60